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# Orientifold's landscape: non-factorisable six-tori

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Abstract: We construct type IIA orientifolds on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ which admit non factorisable lattices. We describe a method to deal with this kind of configurations and discuss how the compactification lattice affects the tadpole cancellation conditions. Moreover, we include D6-branes which are not parallel to O6-planes. These branes can give rise to chiral spectra in four dimensions, thus uncovering a new corner in the landscape of intersecting D-brane model constructions. We demonstrate the construction at an explicit example. In general we argue that obtaining an odd number of families is problematic.

Keywords: Intersecting branes models, Superstring Vacua, D-branes.

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## 1. Introduction

We are living an exciting era for particle physics, in which, the first (definite) results from the LHC (Large Hadron Collider) experiments will put a landmark for this century's particle physics. It is thus natural that investigation of phenomenological aspects of every BSM (Beyond Standard Model) theory be in a flourishing season at this time.

In particular, string theory, as one of the best candidates for a unifying theory, has seen a great deal of development, both in its phenomenological as well as cosmological aspects in recent times. From the phenomenological point of view, D-brane models [1 – 5] are among the most interesting set-ups that have been widely studied in both aspects lately.

More concretely, intersecting D-brane models in toroidal orientifold compactifications, represent a simple yet rich set-up. Particle physics models, with spectra close to that of the Standard Model, both in non-supersymmetric [6 – 16] as well as supersymmetric [17 – 26] configurations, can be constructed (for reviews and more references see [27]).

In this context, most models considered so far in the *landscape*[1] of possibilities, have been restricted to factorisable tori [28].[2] That is, toroidal compactifications where the six dimensional internal manifold, $T^6$, can be factorised as the product of three two-tori $T^6 =$

[1] By landscape here we mean the whole set of possibilities that can arise within string theory constructions. In particular, we are interested in orientifold constructions in type IIA theories.

[2] For statistical investigations of more abstract CFT orientifolds see [29].

$T^2 \times T^2 \times T^2$. However, it is very natural to ask whether more generic toroidal orientifold compactifications can be constructed. A first step in this direction has already been taken in heterotic theory [30 – 35], where orbifold models, which admit more complicated lattices, i.e. non factorisable lattices, were analysed. (An interesting observation is that some of the spectra can be obtained on factorisable tori with the notion of generalised torsion [36].) Also, in the context of Type IIA theory, non-chiral models of orientifolds for supersymmetric $\mathbb{Z}_N$ orbifolds, in non-factorisable tori were constructed in [37]. Using a different language, in [1], a six dimensional $\mathbb{Z}_2$ orientifold of an SO(8) lattice in Type IIB theory was presented.

In the case of factorisable orientifold models, the phenomenological requirement of getting an odd number of families (three) puts a strong constraint on the geometry of the factorisable torus [6]. As it has been shown [6], it is necessary to introduce, besides untilted $T^2$ tori, or type **A** two dimensional lattices [38], also tilted tori, or type **B** lattices.

Pure orientifold (that is, no orbifold action performed) non-supersymmetric models, with spectra very close to that of the Standard Model, were first constructed in [7] along these lines, where one of the three two-tori was tilted. One problem of these non-supersymmetric models, arises from stability issues, due to the presence of NSNS tadpoles and tachyons. A supersymmetric version, in the factorisable orbifold $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$, appeared in [17], where again, only one of the three two-tori was tilted.[3] Such supersymmetric constructions are more under control from the point of view of stability, but contain typically chiral exotic states.

In this note, we take a diversion from the usual factorisable path and explore orientifolds of $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ which admit non-factorisable lattices. In particular, we show that such generalisations are easy to deal with. Moreover, we incorporate parallel as well as non-parallel D6-branes, which can then give rise to four dimensional chiral spectra. We discuss how the tadpole cancellation conditions arise in this more general models and construct an explicit example. Although for phenomenological (and stability) reasons, we focus mainly on four dimensional $\mathcal{N} = 1$ models, we also comment on how non-supersymmetric set-ups can be implemented, which can have interesting phenomenology, in spite of suffering from possible instabilities due to the lack of supersymmetry.

We start in the next section by reviewing the $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold construction in the factorisable case, and fix our notation. We then turn in section 3 to non factorisable models. We consider explicitly the SO(12) root lattice, as an illustrative example of our method of dealing with non factorisable tori. We show how to compute the tadpole cancellation conditions for parallel (to the orientifolds) branes. We then introduce non parallel branes, which are invariant under the orbifold action, and which preserve supersymmetry (although this is not strictly required). We present an explicit model as an example of these constructions.

In section 4, we elaborate on how more general lattices, i.e. non factorisable, put severe constraints on the wrapping numbers. This fact gets then reflected in the intersection numbers, which ultimately are directly connected to the number of families in the mod-

[3] It was shown in [20] that, for phenomenological purposes, introducing two or three tilted tori in these supersymmetric models, provides no solution. Therefore, one has to stick to single tilted tori set-ups.



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els. We show that these constraints give rise generically to an even number of families, irrespective of supersymmetry requirement, if one sticks to orbifold invariant D-branes. We then consider the possibility of having non-invariant branes, and show that whereas supersymmetric models are clearly excluded for giving only even number of generations, non supersymmetric models could still be constructed with spectra close to that of the Standard Model. However, as in the factorisable case, the stability of such constructions is not guaranteed. Finally we present some comments and conclusions in the last section.

## 2. Recap of the factorisable $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold

In this section we rephrase the derivation of known results for $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifolds of factorisable six-tori [39, 40]. Specifically, we perform orientifolds of type IIA strings as in [40]. These are related to the original type IIB model [39] and its generalisations with discrete B-fields [41] by T-duality. The effects of the antisymmetric background tensor in pure orientifold compactifications were first described in [42]. In [43] these results have been extended to the case of $T^4/\mathbb{Z}_N$ orientifolds, while in [44] a more detailed analysis was performed and carried further to four dimensional compactifications as well. Finally, in [45] the connection with type IIA orientifolds through T-dualities was discussed.

When we talk about factorisable six-tori we mean that a decomposition into the product of three two-tori is respected by orbifold and orientifold actions, i.e. each factor is mapped onto itself. It is important to note that the notion of factorisable (or non factorisable) makes sense only in combination with the orbifold and orientifold actions. Further one has to specify the dimensionality of the factors (two in our case). Each of the two-tori can be viewed as a compactification of a complex plane which we parameterise by complex coordinates $z^i$, $i = 1, 2, 3$. We specify the orbifold and orientifold actions by their action on these coordinates. Calling the $\mathbb{Z}_2 \times \mathbb{Z}_2$ generators $\theta$ and $\omega$ we explicitly assign

$$\theta\, z^i = e^{2\pi \mathrm{i}\, v_i}\ z^i \ \ ,\ \ \omega\, z^i = e^{2\pi \mathrm{i}\, w_i}\ z^i, \tag{2.1}$$

with

$$\vec{v} = \left(\frac{1}{2}, -\frac{1}{2}, 0\right) \ \ ,\ \ \vec{w} = \left(0, \frac{1}{2}, -\frac{1}{2}\right). \tag{2.2}$$

Further, the orientifold element $\Omega\mathcal{R}$ acts as world sheet parity inversion $\Omega$ together with complex conjugation on the coordinates

$$\mathcal{R}\, z^i = \overline{z}^i\,,\ \ i = 1, 2, 3. \tag{2.3}$$

To obtain a product of three two-tori we equip each of the three complex planes with a two dimensional compactification lattice. That lattice has to be invariant under orbifold and orientifold actions. The two possible choices are [38]

- the **A** lattice is spanned by $(1, 0)$ and $(0, 1)$ ,
- the **B** lattice is spanned by $(1, 1)$ and $(1, -1)$.

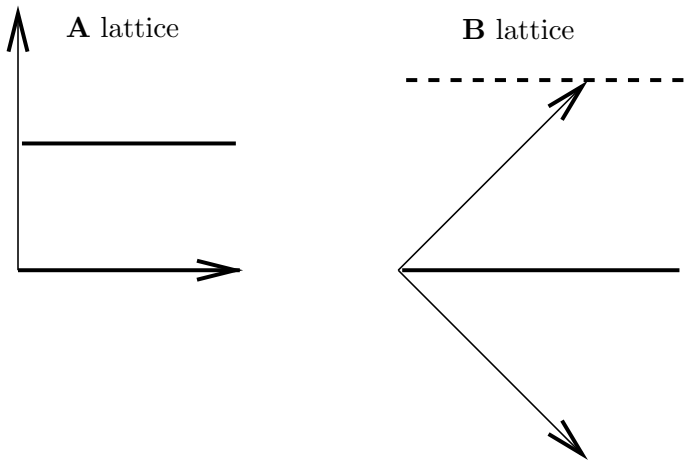


**Figure 1:** The two dimensional compactification lattices: Lattice vectors are drawn with thinner lines than fixed lines. The dashed line in the right figure differs from the thick solid line by a lattice shift. Note, that for the **A** lattice the lengths of the two basis vectors can be different.

Both these compactifications can be decomposed into a product of two circles. Only for the **A** lattice the $\mathcal{R}$-action respects that decomposition. There are two $\mathcal{R}$-fixed lines for the **A** lattice and one for the **B** lattice. The situation is depicted in figure 1. In analogy to our definition of factorisable six-tori, the **A** lattice can be viewed as factorisable into a product of two circles whereas the **B** lattice cannot. It is because of this analogy that the rederivation to be discussed in this section is useful. We use a language which makes a generalisation to non factorisable six-tori (into two-tori) straightforward.

Here, we focus on the case that we introduce D-branes which are parallel to O-planes in order to cancel their RR charges. Each **A** lattice contains twice as many O-planes as the **B** lattice. Therefore each time one replaces an **A** lattice by a **B** lattice the number of D-branes is reduced by a factor 1/2. When we perform the calculation in the string loop channel the number of O-planes does not enter directly. In order to compute the total RR charge one performs a modular transformation to the string tree channel. The zero mode part of the modular transformation consists of Poisson resummations for windings and momenta. This introduces factors depending on the compactification lattice and hence the number of O-planes. In the following we use this method to obtain the anticipated factors of 1/2 for **B** lattices from a loop channel calculation.

First, focus on the Klein bottle and in particular on the contribution with an $\Omega\mathcal{R}$ insertion (other insertions like $\Omega\mathcal{R}\theta$ are straightforward modifications). Windings and momenta need to be invariant under the $\Omega\mathcal{R}$ insertion. This means that momenta are on the $\mathcal{R}$ invariant sublattice of the dual compactification lattice, but this is the dual of an $\mathcal{R}$ projected lattice,[4] which we call $\Lambda^\star_{\mathcal{R},\perp}$.[5] Since $\Omega$ gives an additional sign for

[4]The $\mathcal{R}$ projected lattice is obtained by acting with the operator $(1+\mathcal{R})/2$ on the lattice.

[5]This can be seen by modifying the argumentation in appendix A of [46] (see also [31, 34]). First, we show that every element of $\Lambda^\star_{\mathcal{R},\perp}$ is in the invariant sublattice of the dual lattice $\Lambda^\star$. The projected lattice

windings these are quantised on a $-\mathcal{R}$ invariant lattice which we call $\Lambda_{-\mathcal{R},inv}$. In computing the RR tadpoles one performs a transformation from the string loop channel to the tree channel. For the contributions due to windings and momenta, this implies two Poisson resummations. For each of the $T^2$ factors we obtain a factor (see e.g. footnote 17 in [31])

$$\frac{\mathrm{vol}\,(\Lambda_{\mathcal{R},\perp})}{\mathrm{vol}\,(\Lambda_{-\mathcal{R},inv})}. \tag{2.4}$$

For the **A** lattice this factor is just one whereas for the **B** lattice it is one half (the invariant lattice is generated by (2,0) in this case). So, finally we obtain the RR tadpole contribution from the Klein bottle as (up to some overall constants which we also suppress in the Möbius strip and Cylinder diagrams)

$$\textbf{KB:}\ 2^{-\sum_{i=1}^{3}\delta_{Bi}}32^2, \tag{2.5}$$

where $\delta_{Bi}$ is one (zero) if the $i$th plane is compactified on a **B** (**A**) lattice.

The simplest way of canceling the RR charges of the O-planes is to add D-branes parallel to the O-planes. Let us focus on the set of D6-branes extended along the real axes of the compact space. The open string momenta take values on the dual of the $\mathcal{R}$ invariant lattice $\Lambda^\star_{\mathcal{R},inv}$, whereas windings are transverse to the brane. One subtlety is that the open string has to end only on the same brane but not at the same point. Therefore windings take values on the $-\mathcal{R}$ projected lattice $\Lambda_{-\mathcal{R},\perp}$. For the contribution to the cylinder amplitude with no further insertions into the trace Poisson resummations yield a factor of

$$\frac{\mathrm{vol}\,(\Lambda_{\mathcal{R},inv})}{\mathrm{vol}\,(\Lambda_{-\mathcal{R},\perp})}. \tag{2.6}$$

The relevant contribution to the cylinder amplitude reads

$$\textbf{C:}\ 2^{\sum_{i=1}^{3}\delta_{Bi}}N^2, \tag{2.7}$$

where $N$ is the number of D-branes.

For the Möbius strip we focus again on the contribution with the $\Omega\mathcal{R}$ insertion. The momentum modes are just the same as in the cylinder case, i.e. on $\Lambda^\star_{\mathcal{R},inv}$. However, because $\Omega$ swaps the two ends of the open string it has to end in the same point on the D-brane. The winding modes need to be invariant under $\Omega\mathcal{R}$, i.e. the projected lattice has to be replaced by the invariant one $\Lambda_{-\mathcal{R},inv}$. The two factors appearing due to Poisson

$\Lambda_{\mathcal{R},\perp}$ is three dimensional and so is its dual. Any element of $\Lambda^\star_{\mathcal{R},\perp}$ can be written as $(x,0,y,0,z,0)$, and has integer valued scalar products with every element of $\Lambda_{\mathcal{R},\perp}$. Because the 2nd, 4th and 6th component do not enter, the scalar product with any element of $\Lambda$ is integer valued. Hence, every vector in $\Lambda^\star_{\mathcal{R},\perp}$ lies also in $\Lambda^\star$, and is obviously in its invariant sublattice. It remains to show that also every vector in the invariant sublattice of $\Lambda^\star$ is in $\Lambda^\star_{\mathcal{R},\perp}$. Any $\mathcal{R}$-invariant vector $v\in\Lambda^\star$ satisfies

$$\left(\frac{1+\mathcal{R}}{2}v, q\right)\in\mathbb{Z}$$

for every $q\in\Lambda$. Since $\mathcal{R}$ is symmetric it follows that $v$ has integer valued scalar product with every vector of $\Lambda_{\mathcal{R},\perp}$, and hence is in $\Lambda^\star_{\mathcal{R},\perp}$.



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resummation cancel as well for the **A** as the **B** lattice. The Möbius strip contribution to the RR tadpole reads

$$-2 \cdot 32 \cdot N. \tag{2.8}$$

Adding up the contributions from Klein bottle, Möbius strip and Cylinder one obtains the tadpole cancellation condition

$$2^{\sum_{i=1}^{3} \delta_{Bi}} \left( 32 \cdot 2^{-\sum_{i=1}^{3} \delta_{Bi}} - N \right)^2 = 0, \tag{2.9}$$

which coincides with the result of [40].

For later use we note that we could have just considered the **B** lattice and obtained the **A** lattice result by a modified orientifold action. Instead of changing a lattice from **B** to **A** we can combine the complex conjugation with a multiplication with i:

$$\text{Instead of } \mathbf{B} \to \mathbf{A} \text{ modify } \mathcal{R}\text{: } z^i \to \mathrm{i}\overline{z}^i, \tag{2.10}$$

where $i$ labels the plane in which we want to replace the compactification lattice. Focusing on the corresponding $T^2$ factor, we find that the $\mathcal{R}$ projected lattice is generated by $(1, 1)$ whereas the $-\mathcal{R}$ invariant lattice is generated by $(1, -1)$. The volumes of the two lattices are the same and we obtain, as expected, the same result as if we had considered an **A** compactification. For non factorisable $T^6$, it is convenient, in some cases, to fix the lattice and consider different orientifold actions, instead.

## 3. Non factorisable lattices: SO(12)

As a concrete example of a non factorisable $T^6$, and to exemplify our method, we study a compactification on an SO(12) root lattice with basis vectors:

$$\begin{aligned} e_1 &= (1, -1, 0, 0, 0, 0)\,, \\ e_2 &= (0, 1, -1, 0, 0, 0)\,, \\ e_3 &= (0, 0, 1, -1, 0, 0)\,, \\ e_4 &= (0, 0, 0, 1, -1, 0)\,, \\ e_5 &= (0, 0, 0, 0, 1, -1)\,, \\ e_6 &= (0, 0, 0, 0, 1, 1)\,. \end{aligned} \tag{3.1}$$

Oscillator contributions to amplitudes do not depend on the compactification lattice. The discussion of the previous section can be carried over to non factorisable $T^6$ in a straightforward manner.

### 3.1 Parallel O-planes and D-branes

The orientifold group acts as before on the coordinates in which the lattice vectors (3.1) are given. For the element containing world sheet parity reversal we take as before $\Omega\mathcal{R}$

with $\mathcal{R}$ acting as complex conjugation on the three planes. Then the $\mathcal{R}$ projected lattice is generated by

$$\Lambda_{\mathcal{R},\perp}: \quad \begin{matrix} (1,0,0,0,0,0)\,, \\ (0,0,1,0,0,0)\,, \\ (0,0,0,0,1,0) \end{matrix} \tag{3.2}$$

and has volume one. The $-\mathcal{R}$ invariant lattice is given by SO(6) simple roots

$$\Lambda_{-\mathcal{R},inv}: \quad \begin{matrix} (0,1,0,-1,0,0)\,, \\ (0,0,0,1,0,-1)\,, \\ (0,0,0,1,0,1)\,. \end{matrix} \tag{3.3}$$

The volume of the fundamental cell is the square root of the determinant of the SO(6) Cartan matrix which is two. Hence there is a factor of one half appearing in the Klein bottle amplitude as compared to the factorisable **AAA** compactification. In a very similar way we obtain a factor of two in front of the Cylinder amplitude and conclude that the tadpole cancellation condition is

$$(16-N)^2 = 0. \tag{3.4}$$

This result actually holds for all types of D6-branes parallel to O6-planes (e.g. the consideration of $\Omega\mathcal{R}\theta$, $\Omega\mathcal{R}\omega$ and $\Omega\mathcal{R}\theta\omega$ orientifold fixed planes is completely analogous to the one we carried out, here). So, we obtain the same condition as for e.g. the **BAA** factorisable compactification. Now, however, there is no distinguished complex plane. Actually, we can count the number of O6-planes and confirm that the above result is consistent with the general rule that an O6-plane carries four D6-brane charges. Four inequivalent $\Omega\mathcal{R}$ fixed planes[6] are given by ($z^i = x^i + \mathrm{i}y^i$)

$$\begin{aligned} &\left(x^1,0,x^2,0,x^3,0\right), \quad \left(x^1,\frac{1}{2},x^2,\frac{1}{2},x^3,0\right), \\ &\left(x^1,\frac{1}{2},x^2,0,x^3,\frac{1}{2}\right), \quad \left(x^1,0,x^2,\frac{1}{2},x^3,\frac{1}{2}\right). \end{aligned} \tag{3.5}$$

Note that e.g. the fixed plane $(x,1,y,0,z,0)$ is related by a lattice shift $(1,-1,0,0,0,0)$ to the first plane in (3.5).

If we just focus on two dimensional sublattices the SO(12) lattice looks similar to a **BBB** lattice. In order to point out the difference to the factorisable case, we call our orientifold **CCC** model. As described at the end of the previous section we can obtain different models from the same compactification lattice by replacing the orientifold action according to (2.10). For each plane where such a change is performed we replace a **C** by a **D** in the name of the model.

For instance, we want to change the orientifold action such that we obtain a **DCC** model. We replace the $\mathcal{R}$ action as follows:

$$\mathcal{R}: \quad z^1 \to \mathrm{i}\overline{z}^1, \quad z^i \to \overline{z}^i, \quad i=2,3. \tag{3.6}$$

[6]The treatment of the other O6-planes is completely analogous.

The $\mathcal{R}$ projected lattice is generated by

$$\Lambda_{\mathcal{R},\perp}: \quad \begin{matrix} \left(\frac{1}{2},\frac{1}{2},0,0,0,0\right), \\ (0,0,1,0,0,0), \\ (0,0,0,0,1,0) \end{matrix} \tag{3.7}$$

where the first lattice vector can be obtained by acting with $\frac{1+\mathcal{R}}{2}$ on e.g. $(1,0,0,0,0,1)$. The volume of the fundamental cell is $1/\sqrt{2}$. The $-\mathcal{R}$ invariant lattice is generated by

$$\Lambda_{-\mathcal{R},inv}: \quad \begin{matrix} (1,-1,0,0,0,0), \\ (0,0,0,1,0,-1), \\ (0,0,0,1,0,1) \end{matrix} \tag{3.8}$$

and has volume $\sqrt{8}$. Carrying out a similar consideration for the lattices appearing in the cylinder amplitude one obtains the tadpole cancellation condition

$$(8-N)^2 = 0. \tag{3.9}$$

So, we expect to have two O6-planes per orientifold group element. The two planes for the $\Omega\mathcal{R}$ element are

$$\left(x^1,x^1,x^2,0,x^3,0\right) \quad , \quad \left(x^1,x^1,x^2,\frac{1}{2},x^3,\frac{1}{2}\right). \tag{3.10}$$

Note that $\left(x^1,x^1+1,x^2,0,x^3,0\right)$ is at the same position as the first plane as can be seen by adding the SO(12) lattice vector $(0,-1,1,0,0,0)$. Note also that $\left(x^1,x^1+\frac{1}{2},y,\frac{1}{2},z,0\right)$ is not a fixed plane under the modified action (3.6).

For the **DDC** model we take $\mathcal{R}$ to act as

$$\mathcal{R}: \quad z^i \to \mathrm{i}\overline{z}^i, \quad z^3 \to \overline{z}^3, \quad i=1,2. \tag{3.11}$$

In this case the $\mathcal{R}$ projected lattice is generated by

$$\Lambda_{\mathcal{R},\perp}: \quad \begin{matrix} \left(\frac{1}{2},\frac{1}{2},0,0,0,0\right), \\ \left(0,0,\frac{1}{2},\frac{1}{2},0,0\right), \\ (0,0,0,0,1,0) \end{matrix} \tag{3.12}$$

and its volume is one half. The $-\mathcal{R}$ invariant lattice has the following basis

$$\Lambda_{-\mathcal{R},inv}: \quad \begin{matrix} (1,-1,0,0,0,0), \\ (0,0,1,-1,0,0), \\ (0,0,0,0,0,2). \end{matrix} \tag{3.13}$$

The fundamental cell has volume four. This and similar considerations lead to the tadpole cancellation condition

$$(4-N)^2 = 0. \tag{3.14}$$

Now there is just one $\Omega\mathcal{R}$ fixed plane at

$$\left(x^1,x^1,x^2,x^2,x^3,0\right) \tag{3.15}$$

(replacing the zero by a half does not lead to a fixed plane and adding a one to any of the entries can be mapped to the above plane with a different $x^3$ parameterisation by a lattice shift).

Finally, we consider the **DDD** orientifold, i.e.

$$\mathcal{R}: \quad z^i \to \mathrm{i}\overline{z}^i, \quad i = 1, 2, 3. \tag{3.16}$$

Now, the $\mathcal{R}$ projected lattice is generated by

$$\Lambda_{\mathcal{R},\perp}: \begin{array}{l} \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 0, 0\right), \\ \left(0, 0, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right), \\ \left(\frac{1}{2}, \frac{1}{2}, 0, 0, \frac{1}{2}, \frac{1}{2}\right), \end{array} \tag{3.17}$$

where the first vector is obtained by acting with $\frac{\mathcal{R}+1}{2}$ on the SO(12) root $(1, 0, 1, 0, 0, 0)$. Any $\mathcal{R}$ invariant SO(12) root is on the $\Lambda_{\mathcal{R},\perp}$ lattice as it should be. The volume of $\Lambda_{\mathcal{R},\perp}$ is $1/\sqrt{2}$. The $-\mathcal{R}$ invariant lattice is spanned by

$$\Lambda_{-\mathcal{R},inv}: \begin{array}{l} (1, -1, 0, 0, 0, 0), \\ (0, 0, 1, -1, 0, 0), \\ (0, 0, 0, 0, 0, 1, -1) \end{array} \tag{3.18}$$

and has volume $\sqrt{8}$. Computing volumes of very similar lattices we obtain the tadpole cancellation for the **DDD** orientifold

$$(8 - N)^2 = 0. \tag{3.19}$$

There are indeed two $\Omega\mathcal{R}$ fixed planes in this case

$$\left(x^1, x^1, x^2, x^2, x^3, x^3\right) \quad , \quad \left(x^1 + 1, x^1, x^2, x^2, x^3, x^3\right). \tag{3.20}$$

Note that now the one in the second plane cannot be removed by a lattice shift and a coordinate redefinition.

In summary, we find the tadpole cancellation conditions for the four qualitatively different orientifolds of the SO(12) compactification:

$$\begin{array}{ll} \textbf{CCC}: & (N - 16)^2 = 0\,, \\ \textbf{DCC}: & (N - 8)^2 = 0\,, \\ \textbf{DDC}: & (N - 4)^2 = 0\,, \\ \textbf{DDD}: & (N - 8)^2 = 0\,. \end{array} \tag{3.21}$$

### 3.2 Adding D-branes at angles

So far, we considered having only D6-branes parallel to O6-planes. Open strings stretched between such branes yield non chiral matter. In order to obtain a chiral spectrum we have to add D6-branes forming non-trivial angles with the O-planes.

For simplicity we focus here on branes which are invariant under the orbifold action. (We comment on non invariant branes in section 4.3.) An orbifold invariant D-brane with label $a$ (or stack of $N_a$ D-branes) wraps the three-cycle

$$D6_a = \left(m_a^1\,[a_1] + n_a^1\,[b_1]\right) \times \left(m_a^2\,[a_2] + n_a^2\,[b_2]\right) \times \left(m_a^3\,[a_3] + n_a^3\,[b_3]\right), \tag{3.22}$$

where we used the same notation for one-cycles as in the factorisable literature (e.g. in [17]):

$$\begin{aligned} [a_1] &= (1,0,0,0,0,0)\,, & [b_1] &= (0,1,0,0,0,0)\,, \\ [a_2] &= (0,0,1,0,0,0)\,, & [b_2] &= (0,0,0,1,0,0)\,, \\ [a_3] &= (0,0,0,0,1,0)\,, & [b_3] &= (0,0,0,0,0,1)\,, \end{aligned} \tag{3.23}$$

and $m_a^i$, $n_a^i$ $(i = 1,2,3)$ are integers. The cycle (3.22) is a closed cycle on the SO(12) compactification lattice if

$$m_a^i + n_a^i = \text{even}, \quad i = 1,2,3. \tag{3.24}$$

In all other cases the D-brane has to wrap the cycle (3.22) twice in order to close on the SO(12) root lattice compactification. This can be easiest seen for the case when (3.24) is violated only for one $i$. Then a closed three-cycle is obtained by wrapping the corresponding one-cycle twice. If instead (3.24) is violated for $i = 1,2$ we rewrite,

$$2\prod_{i=1}^{3}\left(m_a^i\,[a_i]+n_a^i\,[b_i]\right) = \left(m_a^1, n_a^1, -m_a^2, -n_a^2, 0, 0\right) \times \left(m_a^1, n_a^1, m_a^2, n_a^2, 0, 0\right) \times \left(0,0,0,0, m_a^3, n_a^3\right), \tag{3.25}$$

where the r.h.s. clearly represents a closed three-cycle in the SO(12) compactified case. (If both, $n_a^3$ and $m_a^3$, are even then (3.25) is actually twice a closed three-cycle.) Eq. (3.25) can be easily verified by employing a one-to-one correspondence of homology and cohomology as discussed, in the present context, e.g. in [47]. Finally, if (3.24) does not hold for any $i$ one notices

$$2\prod_{i=1}^{3}\left(m_a^i\,[a_i]+n_a^i\,[b_i]\right) = \left(m_a^1, n_a^1, -m_a^2, -n_a^2, 0, 0\right) \times \left(m_a^1, n_a^1, m_a^2, n_a^2, 0, 0\right) \times \left(0, 0, m_a^2, n_a^2, m_a^3, n_a^3\right), \tag{3.26}$$

implying that wrapping (3.22) twice suffices to obtain a closed three-cycle on the SO(12) compactification lattice.[7]

In order to compute intersection numbers of two D6-branes one determines a lattice in which the D-branes intersect once. The Jacobian obtained when transforming to the compactification lattice yields the intersection number [37]. To carry out the computation we express the three-cycle (3.22) in terms of SO(12) simple roots (3.1):

$$\begin{aligned} D6_a = &\left(m_a^1 e_1 + \left(m_a^1 + n_a^1\right)\left(e_2 + e_3 + e_4 + \frac{1}{2}e_5 + \frac{1}{2}e_6\right)\right) \times \\ &\left(m_a^2 e_3 + \left(m_a^2 + n_a^2\right)\left(e_4 + \frac{1}{2}e_5 + \frac{1}{2}e_6\right)\right) \times \left(\frac{m_a^3}{2}\left(e_5 + e_6\right) + \frac{n_a^3}{2}\left(e_6 - e_5\right)\right). \end{aligned} \tag{3.27}$$

[7] Similar arguments apply for other lattices. One can express (3.26) in terms of fundamental three-cycles in the SO(12) root lattice. Then imposing co-prime conditions on the resulting expansion coefficients avoids multiple wrappings (counted already in $N_a$).

The lattice in which the brane $D6_a$ and a second brane $D6_b$ intersect once is spanned by the three one-cycles in (3.27) and the corresponding one-cycles of $D6_b$. Thus we obtain for their intersection number $I_{ab}$

$$\begin{aligned} I_{ab} &= \det \begin{pmatrix} m_a^1 & m_a^1+n_a^1 & m_a^1+n_a^1 & m_a^1+n_a^1 & \frac{m_a^1+n_a^1}{2} & \frac{m_a^1+n_a^1}{2} \\ m_b^1 & m_b^1+n_b^1 & m_b^1+n_b^1 & m_b^1+n_b^1 & \frac{m_b^1+n_b^1}{2} & \frac{m_b^1+n_b^1}{2} \\ 0 & 0 & m_a^2 & m_a^2+n_a^2 & \frac{m_a^2+n_a^2}{2} & \frac{m_a^2+n_a^2}{2} \\ 0 & 0 & m_b^2 & m_b^2+n_b^2 & \frac{m_b^2+n_b^2}{2} & \frac{m_b^2+n_b^2}{2} \\ 0 & 0 & 0 & 0 & \frac{m_a^3-n_a^3}{2} & \frac{m_a^3+n_a^3}{2} \\ 0 & 0 & 0 & 0 & \frac{m_b^3-n_b^3}{2} & \frac{m_b^3+n_b^3}{2} \end{pmatrix} \\ &= \frac{1}{2}\prod_{i=1}^{3}\left(m_a^i n_b^i - n_a^i m_b^i\right). \end{aligned} \tag{3.28}$$

In the previous section we identified the three-cycles wrapped by O6-planes and checked that they are consistent with modular transformations. Similar to the D6-branes we can also express the three-cycles wrapped by O6-planes in terms of the (half) cycles (3.23) as,

$$\begin{aligned} O_{\Omega\mathcal{R}} &= (1,0,\text{-}1,0,0,0)\times(0,0,1,0,\text{-}1,0)\times(0,0,1,0,1,0) = 2\,[a_1]\times[a_2]\times[a_3]\,, \\ O_{\Omega\mathcal{R}\theta} &= (0,1,0,1,0,0)\times(0,0,0,\text{-}1,\text{-}1,0)\times(0,0,0,\text{-}1,1,0) = -2\,[b_1]\times[b_2]\times[a_3]\,, \\ O_{\Omega\mathcal{R}\omega} &= (1,0,0,\text{-}1,0,0)\times(0,0,0,1,0,1)\times(0,0,0,1,0,\text{-}1) = -2\,[a_1]\times[b_2]\times[b_3]\,, \\ O_{\Omega\mathcal{R}\theta\omega} &= (0,1,\text{-}1,0,0,0)\times(0,0,1,0,0,1)\times(0,0,1,0,0,\text{-}1) = -2\,[b_1]\times[a_2]\times[b_3]\,. \end{aligned} \tag{3.29}$$

Thus, as far as the RR tadpole cancellation is concerned, we view our brane configuration as a compactification on $\left(T^2\right)^3$ with the number of O-planes doubled (as compared to the SO(12) root lattice). In the following we focus on the **CCC** case discussed in the previous section (see (3.21)). In this case there are half as many O6-planes on the SO(12) lattice compactification as in the **AAA** $\left(T^2\right)^3$ compactification. According to our discussion above we can, hence, just copy the tadpole cancellation conditions from that example [6, 17]:

$$\begin{aligned} \sum_a N_a m_a^1 m_a^2 m_a^3 - 16 &= 0, \\ \sum_a N_a m_a^1 n_a^2 n_a^3 + 16 &= 0, \\ \sum_a N_a n_a^1 m_a^2 n_a^3 + 16 &= 0, \\ \sum_a N_a n_a^1 n_a^2 m_a^3 + 16 &= 0. \end{aligned} \tag{3.30}$$

We look for solutions to (3.30) which preserve $\mathcal{N}=1$ supersymmetry, i.e. the D-branes respect all the supersymmetry which is unbroken by the orbifold and orientifold actions. That is, all D-branes must be related to O-planes by SU(3) rotations commuting with the orbifold group which is in the same SU(3) [49]. For concreteness, we consider different SU(2) subgroups of SU(3) whose centers contain the orbifold elements $\theta$, $\omega$ or $\theta\omega$ (2.1),

| Type | $N_a$ | $m_a^1$ | $n_a^1$ | $m_a^2$ | $n_a^2$ | $m_a^3$ | $n_a^3$ |
|---|---|---|---|---|---|---|---|
| $A_1$ | 6 | 1 | 1 | 1 | −1 | 2 | 0 |
| $B_1$ | 2 | 2 | 0 | 1 | 1 | 1 | −1 |
| $P_1$ | 2 | 0 | 1 | 0 | −1 | 2 | 0 |
| $P_2$ | 6 | 2 | 0 | 0 | 1 | 0 | −1 |
| $P_3$ | 8 | 0 | 1 | 2 | 0 | 0 | −1 |

**Table 1:** D6-brane configuration for a chiral supersymmetric model. As explained in the text, the choice of wrapping numbers $m_a^i$, $n_a^i$ yields one closed three-cycle on the SO(12) lattice compactification, for each type of D6-branes. The number $N_a$ denotes the number of D6-branes in a stack.

| Sector | U(3)×USp(2)×USp(6)×USp(8) | Q |
|---|---|---|
| $A_1 B_1$ | $4\,(3,1,1,1)$ | −1 |
| $A_1\,(P_2 + P_2')$ | $2\,(3,1,6,1)$ | 0 |
| $A_1\,(P_3 + P_3')$ | $2\,(\overline{3},1,1,8)$ | 0 |
| $B_1\,(P_1 + P_1')$ | $2\,(1,2,1,1)$ | −1 |
| $B_1\,(P_3 + P_3')$ | $2\,(1,1,1,8)$ | 1 |

**Table 2:** Chiral spectrum of the configuration in table 1. Here, we adopted the convention that eq. (B2) of [17] yields chiral multiplets in the given representations. $Q$ denotes the charge under the U(1) living on the stack $B_1$. The rules are summarised in appendix A.

respectively. Explicitly, apart from D-branes parallel to O-planes, we allow for three types of D6-branes to be present, if they wrap one of the following three-cycles

$$\begin{aligned} &(a) \qquad (k\,[a_1] + l\,[b_1]) \times (m\,[a_2] + n\,[b_2]) \times [a_3]\,, \\ &(b) \qquad [a_1] \times (k\,[a_2] + l\,[b_2]) \times (m\,[a_3] + n\,[b_3])\,, \\ &(c) \qquad (k\,[a_1] + l\,[b_1]) \times [a_2] \times (m\,[a_3] + n\,[b_3])\,, \end{aligned} \tag{3.31}$$

with

$$\frac{m}{n} = -\frac{k}{l}. \tag{3.32}$$

For consistency one has to add also the orientifold images, i.e. branes for which the signs of $n$ and $l$ are reversed.

Although the tadpole cancellation conditions are rather restrictive one can find a chiral model with several non abelian gauge factors. We list the corresponding D-branes in table 1.

The chiral part of the spectrum is given in table 2 (the detailed rules for computing this spectrum from given intersection numbers can be found in [17], see also appendix A).

The brane configuration in table 1 is suitable to visualise the effect of non factorisable as compared to factorisable compactifications. One can start with the same brane configuration in ten dimensions but instead of compactifying on the non-factorisable SO(12) lattice take the factorisable **AAA** lattice. Now, the branes in table 1 wrap closed three-cycles

| Type | $N_a$ | $m_a^1$ | $n_a^1$ | $m_a^2$ | $n_a^2$ | $m_a^3$ | $n_a^3$ |
|---|---|---|---|---|---|---|---|
| $A_1$ | 12 | 1 | 1 | 1 | $-1$ | 1 | 0 |
| $B_1$ | 4 | 1 | 0 | 1 | 1 | 1 | $-1$ |
| $P_1$ | 4 | 0 | 1 | 0 | $-1$ | 1 | 0 |
| $P_2$ | 12 | 1 | 0 | 0 | 1 | 0 | $-1$ |
| $P_3$ | 16 | 0 | 1 | 1 | 0 | 0 | $-1$ |

**Table 3:** D6-brane configuration for a chiral supersymmetric model in a factorisable **AAA** setting. Now, the choice of wrapping numbers $n_a^i$, $m_a^i$ yields one closed three-cycle on the **AAA** lattice compactification, for each type of D6-branes. In ten dimensions the branes extend along the same directions as the ones in table 1.

| Sector | U(6)×U(2)×USp(4)×USp(12)×USp(16) |
|---|---|
| $A_1B_1$ | $2\left(6,\overline{2},1,1,1\right)$ |
| $A_1\left(P_2+P_2'\right)$ | $1\left(6,1,1,12,1\right)$ |
| $A_1\left(P_3+P_3'\right)$ | $1\left(\overline{6},1,1,1,16\right)$ |
| $B_1\left(P_1+P_1'\right)$ | $1\left(1,\overline{2},4,1,1\right)$ |
| $B_1\left(P_3+P_3'\right)$ | $1\left(1,2,1,1,16\right)$ |

**Table 4:** Chiral spectrum of the configuration in table 3. Comparison with table 2 shows that in the factorisable case the rank of the gauge group factors is twice as big whereas the number of generations is half the numbers obtained for the SO(12) compactification.

twice and one obtains a modified table 3. The corresponding chiral part of the massless spectrum is listed in table 4. We see that replacing a factorisable compactification by a non factorisable one decreases the size of the gauge group whereas it increases the number of generations. Obviously the two models cannot be connected by conventional continuous deformations like spontaneous symmetry breaking by turning on flat directions in the moduli space.

## 4. Number of families

### 4.1 Factorisable lattices

In this section we review the simple argument of why one cannot obtain an odd number of generations on the factorisable **AAA** torus and how introducing a tilted torus solves the problem. On the **AAA** torus the branes wrap the cycles (3.22)

$$D6_a = \left(m_a^1\left[a_1\right]+n_a^1\left[b_1\right]\right)\times\left(m_a^2\left[a_2\right]+n_a^2\left[b_2\right]\right)\times\left(m_a^3\left[a_3\right]+n_a^3\left[b_3\right]\right),$$

where the one-cycles are defined in (3.23), while the image cycles under $\Omega\mathcal{R}$ are obtained

by replacing $n_a^i$ with $-n_a^i$. Hence the intersection numbers are given by

$$\begin{aligned} I_{ab} &= \prod_{i=1}^{3}(m_a^i n_b^i - n_a^i m_b^i), \\ I_{ab'} &= -\prod_{i=1}^{3}(m_a^i n_b^i + n_a^i m_b^i). \end{aligned} \tag{4.1}$$

If the stack $D6_a$ generates the U(3) gauge group and the stack $D6_b$ gives the U(2) gauge group, in order to have three copies of the $(3,2)$ representation of SU(3)$\times$ SU(2) we need either (i) $I_{ab} = 3$ and $I_{ab'} = 0$ or (ii) $I_{ab} = 2$ and $I_{ab'} = 1$. One could also have only the net number of generations to be three, e.g. four generations and one anti-generation. However, from (4.1), it follows that

$$I_{ab} + I_{ab'} = -2\left[m_a^1 m_a^2 n_a^3 n_b^1 n_b^2 m_b^3 + m_a^1 n_a^2 m_a^3 n_b^1 m_b^2 n_b^3 + n_a^1 n_a^2 n_a^3 m_b^1 m_b^2 m_b^3 + n_a^1 m_a^2 m_a^3 m_b^1 n_b^2 n_b^3\right] \tag{4.2}$$

is even for any values of the integer wrapping numbers [6]. The solution proposed in [48] to solve this problem, consists in tilting one of the three two dimensional tori. This amounts to replacing the **A** lattice in one of the tori with a **B** lattice (see section 2)

$$\begin{aligned} e_1 &= (1, -1, 0, 0, 0, 0), \\ e_2 &= (1,\ 1, 0, 0, 0, 0), \\ e_3 &= (0, 0, 1, 0, 0, 0), \\ e_4 &= (0, 0, 0, 1, 0, 0), \\ e_5 &= (0, 0, 0, 0, 1, 0), \\ e_6 &= (0, 0, 0, 0, 0, 1). \end{aligned} \tag{4.3}$$

Expressing the three-cycle (3.22) in terms of the new lattice basis vectors we obtain

$$D6_a = \left(\frac{m_a^1}{2}\,(e_1 + e_2) + \frac{n_a^1}{2}\,(e_2 - e_1)\right) \times \left(m_a^2\, e_3 + n_a^2\, e_4\right) \times \left(m_a^3\, e_5 + n_a^3\, e_6\right), \tag{4.4}$$

which describes a closed cycle on the **BAA** lattice (4.3) if

$$m_a^1 + n_a^1 = \text{even}. \tag{4.5}$$

Equally the intersection number is modified to

$$I_{ab} = \frac{1}{2}\prod_{i=1}^{3}(m_a^i n_b^i - n_a^i m_b^i) \tag{4.6}$$

and therefore, the factor of 2 in front of (4.2) cancels off. Alternatively, the condition (4.5) introduces a minimal factor of 2 if $m_a^1$ and $n_a^1$ are both odd (and a minimal factor of 4 if $m_a^1,\ n_a^1$ are even), which is compensated by the global factor in the intersection number (4.6). Thus, in this case three generations can be obtained in both ways described at the beginning of the section [17, 20] (see table 5 for some examples).



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| Case | Type | $N_a$ | $m_a^1$ | $n_a^1$ | $m_a^2$ | $n_a^2$ | $m_a^3$ | $n_a^3$ |
|---|---|---|---|---|---|---|---|---|
| (i) | $A$ | 6 | 3 | 1 | 1 | $-1$ | 1 | 0 |
| | $B$ | 4 | 1 | 1 | 1 | 0 | 1 | $-1$ |
| (ii) | $A$ | 6 | 1 | 1 | 0 | $-1$ | 1 | 1 |
| | $B$ | 4 | 1 | $-1$ | 3 | 1 | 1 | 0 |

**Table 5:** Examples with three generations on factorisable lattices.

### 4.2 Non factorisable lattices

Let us now consider a lattice similar to (4.3), but which does not factorise under the action of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold

$$\begin{aligned} e_1 &= (1,0,-1,0,0,0), \\ e_2 &= (0,1,0,0,0,0), \\ e_3 &= (1,0,1,0,0,0), \\ e_4 &= (0,0,0,1,0,0), \\ e_5 &= (0,0,0,0,1,0), \\ e_6 &= (0,0,0,0,0,1). \end{aligned} \tag{4.7}$$

The three-cycle (3.22) now takes the form

$$D6_a = \left(\frac{m_a^1}{2}\,(e_1+e_3) + n_a^1\,e_2\right) \times \left(\frac{m_a^2}{2}\,(e_3-e_1) + n_a^2\,e_4\right) \times \left(m_a^3\,e_5 + n_a^3\,e_6\right), \tag{4.8}$$

The intersection number is given again by (4.6), but the condition (4.5) becomes[8]

$$m_a^1\,,\ m_a^2 = \text{even}. \tag{4.9}$$

If these conditions are satisfied, each introduces a minimal factor of two in the intersection number, hence there is a factor of two too many and we cannot obtain odd intersection numbers in this case. If one or both conditions above are violated, however, the situation is similar to the case we studied in section 3.2. That is, the brane has to wrap twice the cycle (3.22). In that case, it is possible to get odd intersection numbers $I_{ab}$. This can be achieved by intersecting branes of two different types. For one type the wrapping numbers $m^1$, $m^2$ are both odd (and one has to wrap the cycle twice), and for the other type both are even. However the total number of families, given by (4.2), is always even for any combination of branes obeying, or not, the conditions (4.9).

In the following we argue that this happens for a general choice of non factorisable lattices. In the example at hand one can see that the condition (4.5) is modified because, in the non factorisable case, the coordinates $x^1$ and $y^1$ are not related anymore in the

[8]Note that in order to keep (4.5) unchanged one would need to consider branes of the form $(m_a^1,0,n_a^1,0,0,0)\times(0,m_a^2,0,n_a^2,0,0)\times(0,0,0,0,m_a^3,n_a^3)$, which are not orbifold invariant. This possibility is discussed in the section 4.3.

basis vectors. Nevertheless the even intersection number problem remains also when the condition (4.5) is preserved.

If we take the example of the SO(12) root lattice, the condition for having closed cycles of the form (3.27) is $m_a^i + n_a^i =$ even, $i = 1, 2, 3$, similar to the factorisable case. If this condition is satisfied for all $i$'s, each contributes with a minimal factor of 2. Thus the single 1/2 factor in the intersection number cannot account for them. As we saw in section 3.2, if this condition is not satisfied for one or all $i$'s, the brane has to wrap the cycle twice. It is again possible to check that, although the single intersection numbers can be odd, the total intersection number $I_{ab} + I_{ab'}$ is always even.

In contrast, in the factorisable case with three tilted tori, i.e. an $SO(4)^3$ factorised lattice,

$$\begin{aligned} e_1 &= (1,-1,0,0,0,0)\,, \\ e_2 &= (1,\ \ 1,0,0,0,0)\,, \\ e_3 &= (0,0,1,-1,0,0)\,, \\ e_4 &= (0,0,1,\ \ 1,0,0)\,, \\ e_5 &= (0,0,0,0,1,-1)\,, \\ e_6 &= (0,0,0,0,1,\ \ 1)\,, \end{aligned} \tag{4.10}$$

although we also have three conditions of the form $m_a^i + n_a^i =$ even, $i = 1, 2, 3$, the intersection number also contains, this time, three factors of 1/2. This can be seen from the intersection number. While in the SO(12) case, equation (3.28), there is only a single 1/2 factor that survives after performing operations that leave the determinant invariant, like adding columns, in the case of the factorised $SO(4)^3$ lattice we have

$$\begin{aligned} I_{ab} &= \det \begin{pmatrix} \frac{m_a^1-n_a^1}{2} & \frac{m_a^1+n_a^1}{2} & 0 & 0 & 0 & 0 \\ \frac{m_b^1-n_b^1}{2} & \frac{m_b^1+n_b^1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{m_a^2-n_a^2}{2} & \frac{m_a^2+n_a^2}{2} & 0 & 0 \\ 0 & 0 & \frac{m_b^2-n_b^2}{2} & \frac{m_b^2+n_b^2}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{m_a^3-n_a^3}{2} & \frac{m_a^3+n_a^3}{2} \\ 0 & 0 & 0 & 0 & \frac{m_b^3-n_b^3}{2} & \frac{m_b^3+n_b^3}{2} \end{pmatrix} \\ &= \frac{1}{8}\prod_{i=1}^{3}\left(m_a^i n_b^i - n_a^i m_b^i\right). \end{aligned} \tag{4.11}$$

And therefore, it is possible to get an odd number of families.[9]

In the non factorisable case, as well, one can obtain a factor of 1/8 in the intersection

[9] Although, as pointed out in [20], other phenomenological requirements eliminate the possibility to get consistent models using more than one tilted torus.

number. Using again an $SO(4)^3$ lattice as an example, we can take, for instance

$$\begin{aligned} e_1 &= (1,0,-1,0,0,0)\,, \\ e_2 &= (1,0,\ \ 1,0,0,0)\,, \\ e_3 &= (0,1,0,0,-1,0)\,, \\ e_4 &= (0,1,0,0,\ \ 1,0)\,, \\ e_5 &= (0,0,0,1,0,-1)\,, \\ e_6 &= (0,0,0,1,0,\ \ 1)\,. \end{aligned} \tag{4.12}$$

In this case, the invariant cycles (3.22) take the form

$$\begin{aligned} D6_a = &\left(\frac{m_a^1}{2}(e_1+e_2)+\frac{n_a^1}{2}(e_3+e_4)\right)\times\left(\frac{m_a^2}{2}(e_2-e_1)+\frac{n_a^2}{2}(e_5+e_6)\right)\times \\ &\left(\frac{m_a^3}{2}(e_4-e_3)+\frac{n_a^3}{2}(e_6-e_5)\right), \end{aligned} \tag{4.13}$$

which translates into the intersection number

$$\begin{aligned} I_{ab} &= \det\begin{pmatrix} \frac{m_a^1}{2} & \frac{m_a^1}{2} & \frac{n_a^1}{2} & \frac{n_a^1}{2} & 0 & 0 \\ \frac{m_b^1}{2} & \frac{m_b^1}{2} & \frac{n_b^1}{2} & \frac{n_b^1}{2} & 0 & 0 \\ -\frac{m_a^2}{2} & \frac{m_a^2}{2} & 0 & 0 & \frac{n_a^2}{2} & \frac{n_a^2}{2} \\ -\frac{m_b^2}{2} & \frac{m_b^2}{2} & 0 & 0 & \frac{n_b^2}{2} & \frac{n_b^2}{2} \\ 0 & 0 & -\frac{m_a^3}{2} & \frac{m_a^3}{2} & -\frac{n_a^3}{2} & \frac{n_a^3}{2} \\ 0 & 0 & -\frac{m_b^3}{2} & \frac{m_b^3}{2} & -\frac{n_b^3}{2} & \frac{n_b^3}{2} \end{pmatrix} \\ &= \frac{1}{8}\prod_{i=1}^{3}\left(m_a^i n_b^i - n_a^i m_b^i\right), \end{aligned} \tag{4.14}$$

but with the conditions $m_a^i = \text{even}$, $n_a^i = \text{even}$, $i = 1,2,3$. Taking into account the possibility of taking non-closed cycles (following the rules discussed in section 3.2) in total, these conditions would introduce a minimal factor of 4 too much. It seems difficult to reduce the number of conditions to one, while maintaining a factor of 1/8, or at least 1/4, in the intersection number.

From the point of view of minimising the factors of 2, conditions of the form (4.5) seem to be preferable. But these conditions seem to be correlated with less global factors of one half (a single factor of 1/2 in the examples above). So, one should try to reduce the number of conditions to one, in a non factorisable way, for instance

$$\begin{aligned} e_1 &= (1,-1,0,0,0,0)\,, \\ e_2 &= (0,1,-1,0,0,0)\,, \\ e_3 &= (0,1,\ \ 1,0,0,0)\,, \\ e_4 &= (0,0,0,1,0,0) \\ e_5 &= (0,0,0,0,1,0)\,, \\ e_6 &= (0,0,0,0,0,1)\,, \end{aligned} \tag{4.15}$$

where

$$D6_a = \left(m_a^1\left(e_1+\frac{e_2+e_3}{2}\right)+\frac{n_a^1}{2}\left(e_2+e_3\right)\right)\times \left(\frac{m_a^2}{2}\left(e_3-e_2\right)+n_a^2 e_4\right)\times\left(m_a^3 e_5+n_a^3 e_6\right). \tag{4.16}$$

The price of realising condition (4.5) in a non factorisable way is to have also $m_a^2 =$ even, while there is still just a single factor of one half in the intersection number. One can think of other examples, but each time the conditions for having closed cycles introduce at least a factor of two too many.

The only other choice one can think of is to consider a different orientifold action, like the one in equation (3.6)

$$\mathcal{R}: \;\; z^1 \to \mathrm{i}\overline{z}^1, \;\; z^i \to \overline{z}^i, \;\; i=2,3.$$

The advantage of this action is that the image branes are not obtained by replacing $n$ with $-n$, but $m$ with $n$ and vice versa, which avoids having even $I_{ab}+I_{ab'}$ in all cases. On the other hand the number of lattices that admit this symmetry is reduced. Particularly interesting in this case are the conditions that restrict only one of the wrapping numbers, say $m_a^i =$ even, $\;i=1,2,3$, since

$$I_{ab'} \sim \prod_{i=1}^{3}\left(m_a^i m_b^i - n_a^i n_b^i\right). \tag{4.17}$$

Actually in order to have conditions only on the wrapping numbers $m$ we need to have a factorised lattice in the coordinates $y^1$, $y^2$ and $y^3$, but the symmetry (3.6) relates the coordinates $x^1$ and $y^1$. So, the condition $m_a^1 =$ even and the (3.6) are not compatible. The other type of conditions, $m_a^i$, $n_a^i =$ even and $m_a^i + n_a^i =$ even, do not make a difference with the previous case ($I_{ab}+I_{ab'}$ is again even).

### 4.3 Non-invariant branes

To complete our search for models in non-factorisable tori, in this section we study the possibility of adding non-invariant branes under the orbifold group. For instance, consider a pair of branes wrapping the following cycles

$$\begin{aligned} D6_a &= (1,0,0,0,0,0)\times(0,0,0,1,3,0)\times(0,0,1,0,0,-1),\\ D6_b &= (1,-1,0,0,0,0)\times(0,0,0,1,1,0)\times(0,0,1,0,0,0). \end{aligned} \tag{4.18}$$

These branes are not invariant under the orbifold action. Moreover, they are rotated with respect to the O6-planes, but along non standard directions. For example, if the six dimensional torus has complex coordinates $z^i = x^i + \mathrm{i}y^i$, $i=1,\,2,\,3$, the first brane above can be put in an invariant form by rotating it by $\pm\pi/2$ in the plane $(x^2,\,x^3)$. Thus such brane would be related to the O6-planes by rotations which do not commute with the orbifold. Therefore, branes of type (4.18), do not preserve any supersymmetry. In spite of

this, one can check that this kind of configurations give rise to an odd number of families. More specifically, from the configuration above one gets $I_{ab} + I_{ab'} = 3$.[10] Thus, even if these configurations do not preserve supersymmetry, one could still construct chiral four dimensional models, which might give spectra close to that of the Standard Model. One would have to check if there is a way to make such models stable or long lived.

## 5. Conclusions

In this note, we have explored in detail orientifold models of Type IIA string theory compactified on non-factorisable lattices. We have concentrated in orientifolds of $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$, which admit more general lattices. In particular, we were interested in lattices that cannot be expressed in a factorisable fashion. Initial work along these lines, was started in [37]. There the authors concentrated in orientifolds of $T^6/\mathbb{Z}_N$, and restricted their study to non-chiral four dimensional models.

We have taken a step further and considered the possibility of including D6-branes at angles, which can then give rise to chiral models in four dimensions. We did this by working explicitly with an illustrative example, the SO(12) lattice. As we saw, once one introduces non-factorisable lattices, the tadpoles conditions change according to the lattice. As expected, lattice vectors forming non trivial angles with Euclidean coordinate axes lead to rank reductions in the gauge symmetries. Moreover, we saw that consistency with the compactification imposes strong constraints on the wrapping numbers of the D6-branes, $(m, n)$. These conditions get reflected in the intersection numbers, which are directly connected to the number of families. As we showed, when one considers orbifold invariant branes, the total number of families, which is given by $I_{ab} + I_{ab'}$ turns out to be always even, whether supersymmetry is imposed or not.

In the case of non-invariant branes, it is supersymmetry which forbids to get an odd number of families. However, as we saw in the last section, non-supersymmetric models with odd number of families can be constructed, although their stability might be a problematic issue.

Thus, our findings seem to imply a dramatic conclusion. The models in [17] appear to be, as the authors stated, quite unique. Unfortunately, all those models suffer from the presence of several chiral exotic particles in their spectra. Therefore, one would be tempted to conclude that supersymmetric orientifold models on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ are not viable phenomenologically. A possibly related observation has been reported in [50]. In an approach along the lines of [29], with the CFT given by free fermions, the collaboration could exclude all models phenomenologically. It is not clear that there should be a connection to our results. In the context of heterotic constructions, it has been conjectured, however, that semi realistic free fermionic models and orbifolds of non factorisable six-tori are related [51]. Therefore, there might be some correlation between our results and those of [50].

[10] This can be realised on a lattice similar to (4.7), but with the non factorisable lattice in the coordinates $y^2$ and $x^3$, instead of $x^1$ and $x^2$. Intersection points which are not invariant under the orbifold are mapped onto intersection points of image branes.

It would be interesting to explore chiral model constructions in other orientifolds, for instance those discussed in [37], to see if the situation can be ameliorated. A possibility to improve the situation for $\mathbb{Z}_2 \times \mathbb{Z}_2$ may be to include projections acting geometrically as free shifts as described in [52] for heterotic orbifolds and in [53] for orientifolds.

Another issue that we did not touch at this level of our discussion, is the problem of moduli stabilisation. In the context of factorisable tori, this issue has been investigated in [54]. However, as in the case of [17], such models have chiral exotic fields.


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## A. Model building rules

Here, we summarise the model building rules for the $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold. A stack of $N$ D6-branes not situated on top of an O6-plane accommodates the gauge symmetry $U(N/2)$. If $N$ D6-branes are located on top of an O6-plane they give rise to the gauge factor $USp(N)$. The chiral spectrum comes from strings stretched between branes tilted with respect to the O-planes. The rules are [6] ($a \neq b$):

- Strings stretching between the brane-stacks $N_a$ and $N_b$ give rise to $I_{ab}$ multiplets in the $\left(\frac{N_a}{2}, \frac{\overline{N}_b}{2}\right)$ representation of $U(N_a/2) \times U(N_b/2)$.
- Strings stretching between the brane-stack $N_a$ and the $\mathcal{R}$-image-stack $N_{b'}$ yield $I_{ab'}$ multiplets in the $\left(\frac{N_a}{2}, \frac{N_{b'}}{2}\right)$ representation of $U(N_a/2) \times U(N_{b'}/2)$.
- Strings stretching between the brane-stack $N_a$ and its $\mathcal{R}$ image provide $\frac{1}{2}(I_{aa'} + 4I_{aO6})$ multiplets in the anti-symmetric representation of $U(N_a/2)$, and $\frac{1}{2}(I_{aa'} - 4I_{aO6})$ in the symmetric representation. Here, O6 refers to the sum of all three-cycles wrapped by O6-planes.

In addition there is non-chiral matter.

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